Quiz 5, Calculus III

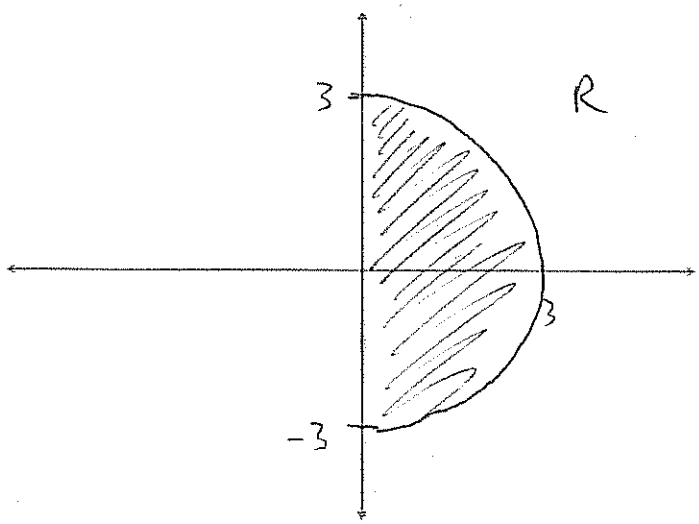
Fall 2012

7:44
7:52
8 min
⇒ give 20→25

Name: Key

1. (3 points) Sketch the region of integration, then evaluate the iterated integral. Change the coordinate system if you think it will simplify the integral.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 4x \, dy \, dx$$



$$\int_0^3 \left(4xy \Big|_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \right) dx$$

$$= 4\int_0^3 x\left(\sqrt{9-x^2} - (-\sqrt{9-x^2})\right) dx$$

$$= 8\int_0^3 x\sqrt{9-x^2}\, dx$$

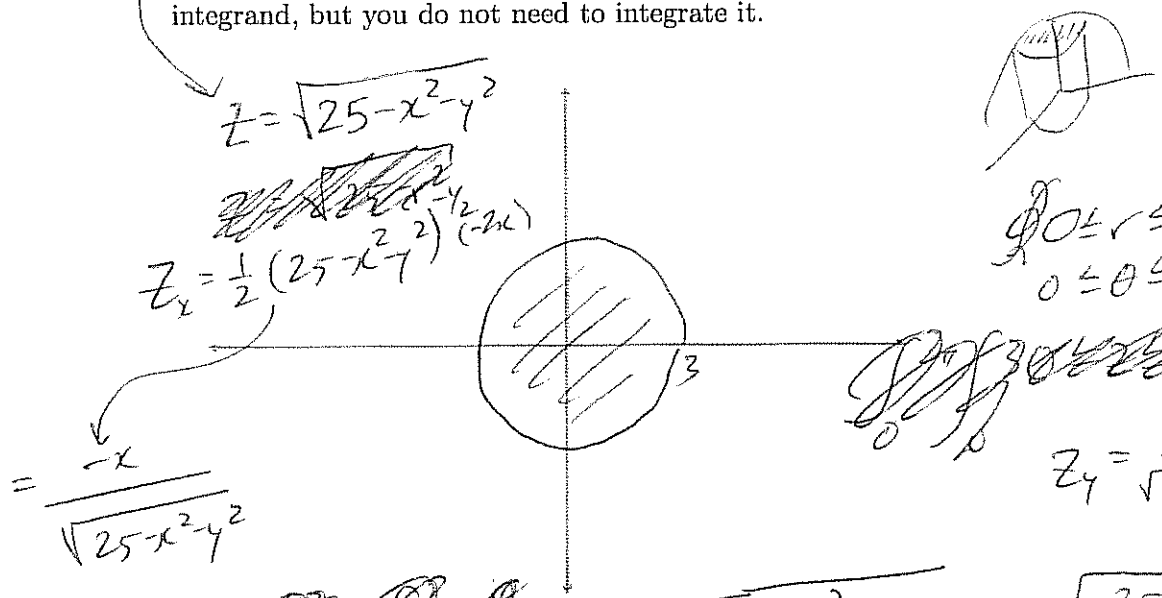
$u = 9-x^2$ $\quad 0 \to 9$, $\quad 3 \to 0$

$du = -2x\,dx$

$$= -4\int_9^0 u^{1/2}\, du$$

$$= -4 \cdot \frac{2}{3} u^{3/2}\Big|_9^0 = \frac{8}{3}(27) = \boxed{72}$$

2. (3 points) Set up, but do not integrate, an integral to find the surface area of upper portion of the sphere $x^2 + y^2 + z^2 = 25$ that lies within the cylinder $x^2 + y^2 = 9$. If the integral needs to be converted to a different coordinate system, do the conversion and simplify the integrand, but you do not need to integrate it.



$z = \sqrt{25 - x^2 - y^2}$

$z_x = \frac{1}{2}(25 - x^2 - y^2)^{-1/2}(-2x)$

$= \frac{-x}{\sqrt{25 - x^2 - y^2}}$

$0 \le r \le 3$

$0 \le \theta \le 2\pi$

$z_y = \frac{-y}{\sqrt{25 - x^2 - y^2}}$

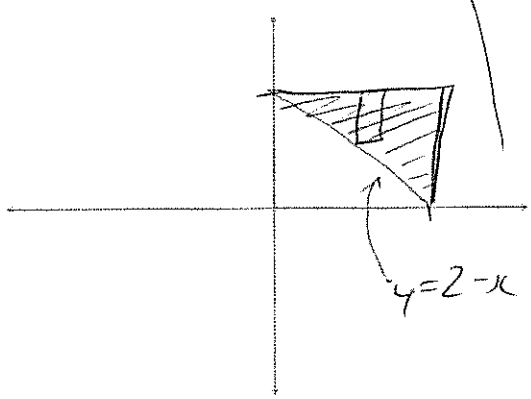
$\iint_D \sqrt{1 + \frac{x^2}{25 - x^2 - y^2} + \frac{y^2}{25 - x^2 - y^2}} = \sqrt{\frac{25 - x^2 - y^2 + x^2 + y^2}{25 - x^2 - y^2}}$

$= \frac{5}{\sqrt{25 - x^2 - y^2}}$

$\Rightarrow \int_0^{2\pi} \int_0^3 \frac{5}{\sqrt{25 - r^2}}\, r\, dr\, d\theta$

$\Leftarrow$ Moonspace

3. (4 points) Set up, but do not integrate, a triple integral to find the volume of the following solid: The solid lies under the surface $z = 2xy$ and lies above the triangle in the xy-plane that has vertices (2,0), (0,2), and (2,2).



$\int_0^2 \int_{2-x}^2 \int_0^{2xy} dz\, dy\, dx$